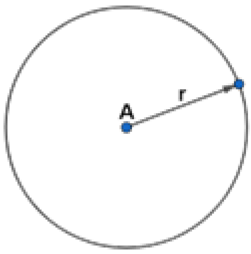
Revision Notes

Class 10 Mathematics

Chapter 12 – Areas Related to Circles

Circles:

The path of a point moving in such a way that its distance from a fixed point is always the same is called a **circle**. That fixed point is called the **centre** of that circle and that path is called the **locus** of that point. The fixed distance between the centre and path is called the **radius** of that circle. We can see many examples of circles around us like bangles, round chapatis, dial watch, sun, etc.



- **Perimeter of a circle –** Perimeter is the **circumferential length** of a closed shape or a polygon. In case of circle, if we travel once around a circle, then the length covered gives us the perimeter of circumference. Circumference of a circle always bears a constant ratio with its diameter, which is denoted by a

  Greek letter π**.** Mathematically,

  $$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

  $$\Rightarrow \text{Circumference} = \pi \times \text{diameter}$$

  $$\Rightarrow \text{Circumference} = \pi \times d$$

  $$\Rightarrow \text{Circumference} = \pi \times 2r$$ (Where r is the radius of circle and $d = 2r$).

- **Area of a circle –** The space covered or occupied by a polygon in a **two-dimensional plane** is called the **area**. In case of a circle, it is the space occupied withing its boundary or the perimeter. If, r is the radius of a given circle, then the formula for finding the area is given as;

  $$\text{Area} = \pi \times r^2$$

Pi (π):

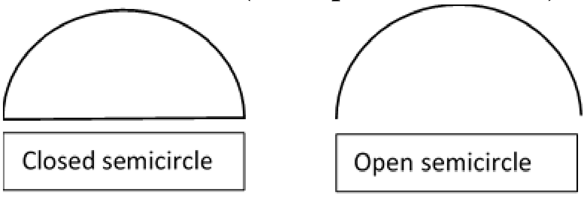
The value of π was given by the great **Indian mathematician Aryabhatta**. He gave an approximate value of π as $\pi = \frac{62832}{20000}$ which is almost equal to 3.1416. It should be noted that π is an irrational number as its value is **non-terminating** and **non-recurring**. For calculation purposes, we often take the value of π as $\frac{22}{7}$ which in turn is a rational number.

Semicircle:

When a circle is cut into half along a diameter, semicircle is formed as shown below. Its perimeter consists of length of half a circle and the length of a diameter. If the semicircle is open, then diameter length is not added. If the length of diameter is given by d and radius is given by r then perimeter is given by,

$\text{Perimeter} = \pi r + d$ (For closed semicircle)

$\text{Perimeter} = \pi r$ (For open semicircle)

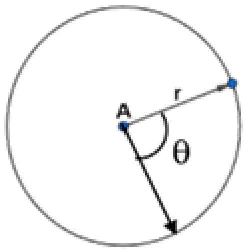


And the area of a semicircle is just half the area of a circle and is mathematically given as $\frac{\pi r^2}{2}$.

Similarly, area of a quadrant of a circle is given by $\frac{\pi r^2}{4}$.

Sector of a circle:

The portion of a circle enclosed within an arc and two radii of that circle is called as **sector**.



Let us take the central angle between the radii is θ which is 360° for a complete circle. Now let the length of that arc be l. Then the length l can be found out using the following relation,

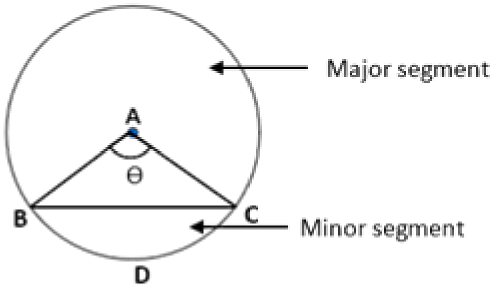
$l = \frac{\theta}{360^\circ} \times 2\pi r$.

Now, perimeter of sector is given as $2r + l$.

Similarly, area of sector is given by $\frac{\theta}{360^\circ} \times \pi r^2$.

Segment of a circle:

The part of the circular region enclosed between a chord and the corresponding arc of that circle is called the **segment** of a circle. The chord having centre of the circle as a point on it is the diameter and also the longest chord of the circle and divides the circle into two equal halves. When the chord is not the diameter, then the portion consisting the centre of circle is called the **major segment** and the other region is called the **minor segment**.



In the diagram above the chord, BC divides the circle in two segments. Such as;

Area of minor segment $=$ Area of sector $ABDC -$ Area of ΔABC.

And area of major segment $=$ Area of circle $-$ Area of minor segment.

Here, area of ΔABC can be found out using the formula $\frac{1}{2} r^2 \sin\theta$.

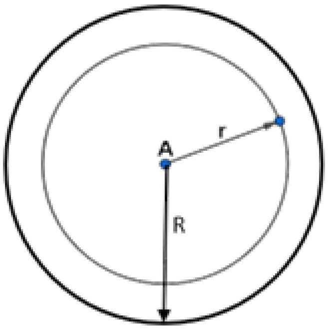
And the area od sector $ABDC$ is given by $\frac{\theta}{360^\circ} \times \pi r^2$.

Hence, the area of segment $ACB = \left(\frac{\theta}{360^\circ} \times \pi r^2\right) - \left(\frac{1}{2} r^2 \sin\theta\right)$

$= r^2 \left[\frac{\pi\theta}{360^\circ} - \frac{\sin\theta}{2}\right]$.

Area of a ring:

Ring is the region between two concentric circles having different radii. Let the radius of larger circle be R and radius of smaller circle be r.



Hence the area of the ring is given by;

$$\pi R^2 - \pi r^2$$

$$= \pi\left(R^2 - r^2\right).$$